Financial Frictions - exam solutions (Feb 17, 2016)

General remarks

Please note that the maximum possible grade of the exam is 180. The scaling grade was used to match minutes and points and thus guide students in the use of time.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that deposit withdrawals are negative).

1. True. This is a channel for the transmission of monetary policy shocks that incorporates the effects of asymmetric information between a bank and its investors, thus forcing the bank to reduce lending after a contractionary monetary policy. For this channel to be active it is required that bank loans and market finance be imperfect substitutes for borrowers, since otherwise a bank that has to reduce lending would not affect a firm's real decision as it can instead raise the same funds through markets.

2. False. Mian and Sufi (2011) study the behavior of existing homeowners, i.e. those that already owned a home by 1997. For these it is found that they increase home equity borrowing for consumption reasons.

3. False. After a negative shock borrowers have to fire sale capital. And their leverage is given by the ratio of capital prices to the required downpayment to buy capital, $L = \frac{q_t}{q_t - \frac{q_{t+1}}{1+r}}$. Since $q_t < q_{t+1}$, while in steady state $q_t = q_{t+1}$, leverage is higher after the shock than in steady state and is thus countercyclical.

4. a) The first best in this economy happens when either p is observable, or when $w > 1 + C$. In this case entrepreneurs after learning the value of p decide to invest if:

$$pX \geq 1 + r.$$

There is a marginal value, p^*, such that only those projects with $p \geq p^*$ are financed. Let us now see when it is worth to pay consultants C to learn p. Doing so gives entrepreneurs the option to make the investment if it is profitable, or to invest in the riskless asset otherwise. Thus, the benefit of knowing p is :

$$V = E[\max(pX, 1+r)] - (1+r) = E[pX - (1+r)|p \geq p^*],$$

Under the assumption that $V > C$, all entrepreneurs will hire consulting services.

Since the distribution function of entrepreneurs' project quality is $F(p)$, in the first best investment, I, and output, q, are given by:

$$\begin{aligned} I^* &= \mu(1 - F(p^*)), \\ q^* &= 1 + r + \mu V. \end{aligned}$$

Note: since consulting fees are income for consultants, the output is given by the risk free rate plus the gains for entrepreneurs, $V - C$, times how many entrepreneur are in the economy, μ, plus the gains for consultants, μC. This is the difference with the Bernanke and Gertler model seen in class where C was a non-pecuniary cost for entrepreneurs only.

b) Now outside investors cannot observe p and entrepreneurs need outside funding. After paying a consultant C to learn p, entrepreneurs with wealth w must decide whether to borrow $1 - w$, knowing that the debt contract provides for a repayment of $R(w)$ if the project is successful. They will borrow if they find it profitable, i.e. if,

$$(X - R(w))p \geq (1+r)w$$

There is a probability of success, $\hat{p}(w)$ which leaves entrepreneurs indifferent, which corresponds to the case of the previous inequality holding as an equality.

c) The condition of zero profits for financiers that lend funds to entrepreneurs of wealth w is given by

$$E[p|p > \hat{p}(w)]R(w) = (1+r)(1-w).$$

Competition will drive away any financier trying to charge a higher payoff if the project is successful.

d) As done for the first best, we can calculate what is benefit for an entrepreneur with wealth w of paying the cost C to learn the probability of success of her project (last step uses zero profit condition for financiers):

$$\begin{aligned}
V(w) &= E[\max[p(X - R(w)), (1+r)w]] - (1+r)w \\
&= \int_{\hat{p}(w)}^{1} [p(X - R(w)) - (1+r)w] dF(p) \\
&= \int_{\hat{p}(w)}^{1} [pX - (1+r)] dF(p).
\end{aligned}$$

To show $V(w)$ is increasing in w we start with the system of two equation in the two unknowns $\hat{p}(w)$ and $R(w)$:

$$\begin{aligned}
(X - R(w))\hat{p}(w) &= (1+r)w, \\
E[p|p > \hat{p}(w)]R(w) &= (1+r)(1-w).
\end{aligned}$$

Taking derivatives with respect to w on both equations, and eliminating the term $\frac{dR(w)}{dw}$ we get

$$\left\{ E[p|p > \hat{p}(w)][X - R(w)] + \hat{p}(w) \frac{dE[p|p > \hat{p}(w)]}{d\hat{p}(w)} \right\} \frac{d\hat{p}(w)}{dw} = (1+r) \left[E[p|p > \hat{p}(w)] - \hat{p}(w) \right].$$

Since $E[p|p > \hat{p}(w)] - \hat{p}(w) > 0$, $\frac{dE[p|p>\hat{p}(w)]}{d\hat{p}(w)} > 0$, and $X - R(w) > 0$, it must be that $\frac{d\hat{p}(w)}{dw} > 0$. Now we can calculate $\frac{dV(w)}{dw}$

$$\frac{dV(w)}{dw} = -[\hat{p}(w)X - (1+r)] f(\hat{p}(w)) \frac{d\hat{p}(w)}{dw} > 0.$$

The sign follows since $\frac{d\hat{p}(w)}{dw} > 0$ implies $\hat{p}(w) \leq p^*$ (equality trivially follows for $\hat{p}(1+C)$ since there is no need to borrow and thus we have the first best), thus making $\hat{p}(w)X - (1+r) < 0$.

e) In this economy investment and output are given by:

$$\begin{aligned}
I &= \mu \int_{w^C}^{\infty} (1 - F(\hat{p}(A))) dH(w), \\
q &= 1 + r + \mu \int_{w^C}^{\infty} V(w) dH(w).
\end{aligned}$$

It is clear that the effect on output is negative, since both the number of entrepreneurs who pay the cost C, as the value they gain from knowing the probability of success of their projects, fall. The effect on investment is ambiguous, because although there are fewer entrepreneurs who pay the cost C, those who pay invest more than what is optimal.

f) Those entrepreneurs that pay the consulting fee, and the financiers lending to them, know that with some probability they will be investing in a negative NPV project (remember $E[p]X < 1 + r$), thus they will be more discriminating than before. Thus, $\hat{p}(w)$ increases. Because now with some probability entrepreneurs are investing in negative NPV projects, the value of "knowing" p is lower than before, i.e. $V(w)$ is reduced. Thus, the information shock will deter marginal entrepreneurs from paying the consulting fee, thus w^C increases. These effects reduce investment (note that investment could still be higher than first best), and output.

5. a) The first best solves:

$$\begin{aligned} \max_{I, c_1^k, c_2^k} \quad & \sum_{k=L,H} p_k \left[\pi_k \ln(c_1^k) + (1 - \pi_k) \ln(c_2^k) \right], \\ s.t. \quad & \sum_{k=L,H} p_k \pi_k c_1^k = 1 - I, \\ & \sum_{k=L,H} p_k (1 - \pi_k) c_2^k = RI. \end{aligned}$$

Since the optimal allocation must completely insure depositors from aggregate liquidity shocks at the level of their individual banks, $c_1^k = c_1^*$, $c_2^k = c_2^*$, and the solution is given by:

$$c_1^* = \frac{1 - I}{\pi_a}, \qquad c_2^* = \frac{IR}{1 - \pi_a}, \qquad k = L, H$$
$$\pi_a = p_L \pi_L + p_H \pi_H = p_H \pi_H.$$

Since utility is logarithmic the first best will then satisfy

$$\frac{1}{c_1^*} = \frac{R}{c_2^*},$$

using the intertemporal budget constraint this gives $c_1^* = 1$, $c_2^* = R$, and $I^* = 1 - p_H \pi_H$.

b) If banks are forced to offer contracts and honor them without interacting with other banks, then they must offer contingent deposit contracts:

$$c_1(\pi) = \frac{1 - I}{\pi}, \qquad c_2(\pi) = \frac{IR}{1 - \pi}.$$

Note that since $L = 0$ no other better contract can be offered, and consumers suffer all the aggregate liquidity risk from their banks. In this case the best contract solves

$$\max_I p_L \ln(IR) + p_H \left[\pi_H \ln\left(\frac{1-I}{\pi_H}\right) + (1-\pi_H)\ln\left(\frac{IR}{1-\pi_H}\right)\right]$$

The first order condition is

$$\frac{1-p_H\pi_H}{I} = \frac{p_H\pi_H}{1-I}.$$

Due to the logarithmic preferences it is the case that optimal investment in autarky is the same as in the first best: $I = 1 - p_H\pi_H$.

c) This allocation can be implemented in a decentralized way through an interbank market. Banks of type L have excess liquidity $M_L = 1 - I^* - \pi_L c_1^* = 1 - I^*$, while banks of type H have liquidity needs $M_H = \pi_H c_1^* - (1 - I^*)$. In the aggregate supply and demand match (follow from optimal quantities in a)):

$$p_L M_L = p_H M_H.$$

To find the equilibrium interest rate in the interbank market we look at transactions in period $t = 2$. Banks of type H have excess liquidity that they use to pay their interbank loans. The interbank rate, $1+r$, derives from equating this payment with $(1+r)M_H$, i.e.:

$$(1+r)M_H = RI^* - (1-\pi_H)c_2^*.$$

This gives:

$$1 + r = \frac{\pi_a}{1-\pi_a}\frac{I^*}{1-I^*}R = R.$$

Again, the assumption of logarithmic preferences gives a simple solution.

d) If banks' liquidity shocks are not observable, then banks of type L would have incentives to pose as H if $r < R - 1$, and banks of type H would have incentives to pose as L if $r > R - 1$ (in both cases to profit from an arbitrage opportunity). Since in this setup $1 + r = R$ no bank has an incentive to lie and the interbank market allocation is incentive compatible. (note that if this were not the case, the interbank allocation would have to be distorted up to the point that $1 + r = R$, this is the condition that must be satisfied in general for the interbank interest rate when shock are unobservable)

e) If consumers are allowed to trade a bond at $t = 1$ when the uncertainty about type is revealed, this allows to transfer consumption from the patient to the impatient at $t = 1$ without the need to inefficiently liquidate investments. Let the price of the bond be $p \leq 1$.

Budget constraints are given by:

$$\begin{aligned} c_1 &= 1 - I + pRI, \\ c_2 &= \frac{1-I}{p} + RI = \frac{C_1}{p}. \end{aligned}$$

Equilibrium requires $p = \frac{1}{R}$ (otherwise demand would not equal supply in the bond market), thus the interest rate on the bond is R, and $c_1 = 1 = c_1^*$ and $c_2 = R = c_2^*$. Which, again due to logarithmic preferences, coincides with the first best. Note that for this to be an equilibrium all consumer have to choose $I = I^* = 1 - p_H \pi_H$.

f) Social welfare is the same in a) and c) since an interbank market can implement the first best. Welfare in d) and e) is the same as in c) since logarithmic preferences imply that the bond market allocation is the first best, and also imply that the interbank interest rate is equal to R eliminating incentives to misbehave if there is private information. Welfare in b) is lower since in this case consumers are exposed to liquidity risk.